Int. J. Multiphase Flow, Vol. 1, pp. 753–769. Pergamon/Elsevier, 1975. Printed in Great Britain.

VELOCITY OF SOUND IN TWO-PHASE MEDIA

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(*Received* 10 *March* 1974)

Abstract—A theory is developed for calculating the velocity of sound in solid–gas and liquid–gas systems. Using mass and momentum balances, an equation is obtained which shows that the velocity of sound depends on the relative velocity between the two phases, on the ratio of densities, on the porosity, on the particle diameter, on the drag coefficient, and on the frequency of sound.

The theory can be applied only if the one phase is distributed in the other homogeneously in the form of particles, droplets or bubbles of equal size. The experimental results agree very well with the theory.

Based on Hertz's equation for the deformation of spheres, a further theory is described which enables calculation of the velocity of sound in gas–solid systems of low porosity, such as packings or porous bodies.

1. INTRODUCTION

Sonic and ultrasonic waves are finding wide application in the chemical and process industries because of their beneficial effects. The following examples may be given: improvement of the stability of fluidized beds (Morse 1955; Molerus 1967), improvement of the flow properties of fine materials (Medcraft 1971), and as a means of coagulating fine mists by facilitating their separation (Green & Lane 1964, Podol'skii & Turubarov 1966). Furthermore, there are many technical problems associated with the interaction of sonic waves and two-phase-media; these occur in pneumatic transport (Weber 1973), the spreading of oil-droplets in burners (Scholz 1972), jet milling (Muschelknautz & Rink 1971), the determination of Young's modulus in bulk materials (Meister 1968) and the measurement of porosity in fluidized beds (Grek & Kiselnikov 1964). The geophysical investigation of rocks, sands and other materials (Wachholz 1962) and the problem of the propagation of sound in structures (Cremer & Heckl 1967) are also subjects whose proper understanding requires the investigation of the general behaviour of sonic waves in multiphase systems.

The attenuation of sound has been investigated by many authors. Rayleigh (1877) and Sewell (1910) published the first papers in this field; more recently Epstein & Carhardt (1953), Goldman (1970) and Soo (1960) have put forward more detailed and precise theories. Many other authors have contributed to this problem but the attention of the present paper is confined to the velocity of sound in two-phase media. Less work has been done in this field: Zink & Delasso (1958), Temkin & Dobbins (1966), and Soo (1967) are almost the only authors who have dealt with this problem. Their theories consider viscous interaction and heat transfer in a system of fine solid particles suspended in a gas, the relative velocity between the phases being overlooked. Other authors like Fischer (1967) and Deich *et al.* (1964) define an average density of gas and solid phase and calculate the velocity of sound by using the well-known Laplace's equation (Gerthsen 1971), which is only valid for a unique phase.

Recently, Rumpf & Gregor (1973) investigated the velocity of sound in a solid–gas mixture flowing in the direction of sound propagation. Applying mass and momentum balances to the advancing pressure wave, they found the sonic velocity to depend on the porosity, on the density-ratio, on the relative velocity and on the relative acceleration between the phases. To a first approximation the latter was found to be a function of the particle diameter, the drag coefficient and the sound frequency.

2. VELOCITY OF SONIC-WAVE PROPAGATION IN A TWO-PHASE SUSPENSION

A suspension containing two different phases, F and S, is considered. The velocity of phase F is given by v, the velocity of phase S by w; thus the relative velocity between the two phases is:

$$v_{\mathrm{rel}} = v - w. \quad [1]$$

The respective densities of the compressive phases are ρ_f and ρ_s. The phase S is homogeneously distributed in the phase F. The porosity ε is defined as the ratio of the area occupied by phase F, to the total cross-sectional area. If a small pressure wave with an absolute velocity a passes through this suspension, then the velocities, densities, and the porosity are changed as shown in figure 1a, wherein no thermodynamic effects are considered, because, as already pointed out by Soo (1967), the dispersion of sound is mainly determined by the viscous interaction between the two phases and the effect of heat transfer can therefore be neglected. If the element considered is moving with the same velocity a as the advancing pressure wave, a steady state is reached (figure 1b). The velocity of sound a_{fs} in a suspension is defined as the velocity of the pressure wave relative to the motion of phase F:

$$a_{fs} = a - v. \quad [2]$$

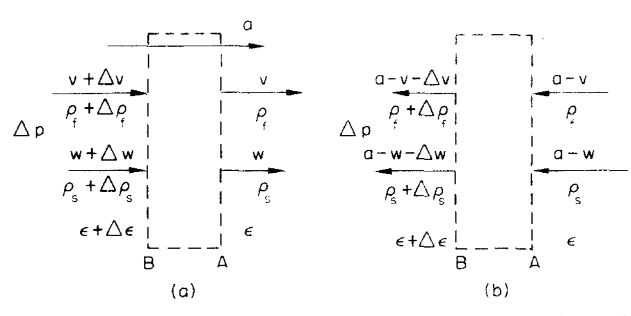


Figure 1. Propagating pressure wave with (a) fixed volume element; and (b) moving volume element (steady state).

In accordance with figure 1b mass balances for the phases F and S yield

$$(a - v)\cdot \varepsilon \cdot \rho_f = (a - v - \Delta v)(\rho_f + \rho_f)(\varepsilon + \Delta\varepsilon)$$
$$(a - w)(1 - \varepsilon)\rho_s = (a - w - \Delta w)(\rho_s + \Delta\rho_s)(1 - \varepsilon - \Delta\varepsilon) \qquad [3]$$

and the momentum balance gives:

$$p + (a - v - \Delta v)^2(\rho_f + \Delta\rho_f)(\varepsilon + \Delta\varepsilon) + (a - w = \Delta w)^2(\rho_s + \Delta\rho_s)(1 - \varepsilon - \Delta\varepsilon)$$
$$= (a - v)^2\varepsilon\rho_f + (a - w)^2(1 - \varepsilon)\cdot\rho_s \qquad [4]$$

wherein each term has the dimension of momentum flux per unit cross-sectional area.

The velocity of sound in the pure phases F and S is given by Laplace's equation:

$$a_f = \sqrt{\frac{\mathrm{d}p}{\mathrm{d}\rho_f}}, \qquad a_s = \sqrt{\frac{\mathrm{d}p}{\mathrm{d}\rho_s}}. \qquad [5]$$

Replacing the differentials in [5] with the ratios of the corresponding increments, and using [1]–[5], the following expression for the velocity of sound a_{fs} in a suspension may be obtained:

$$a_{fs}^2\left(\frac{1}{a_f^2} + \frac{\rho_f}{\rho_s}\frac{1-\varepsilon}{\varepsilon}\frac{1}{a_s^2}\right) = \frac{1 + \dfrac{1-\varepsilon}{\varepsilon}\cdot\dfrac{a_{fs}}{a_{fs} + v_{\mathrm{rel}}}\cdot\dfrac{\Delta w}{\Delta v}}{\varepsilon\left(1 + \dfrac{1-\varepsilon}{\varepsilon}\cdot\dfrac{a_{fs} + v_{\mathrm{rel}}}{a_{fs}}\cdot\dfrac{\rho_s}{\rho_f}\cdot\dfrac{\Delta w}{\Delta v}\right)}. \qquad [6]$$

An unknown value in [6] is the relative acceleration $\Delta w/\Delta v$ of both phases.

For determining this term the volume element of the suspension, to which the mass and momentum balances have been applied, is split into one part for phase F and into another for phase S. Then the drag force between both phases W_{fs} related to the total cross-sectional area A has to be considered (figure 2).

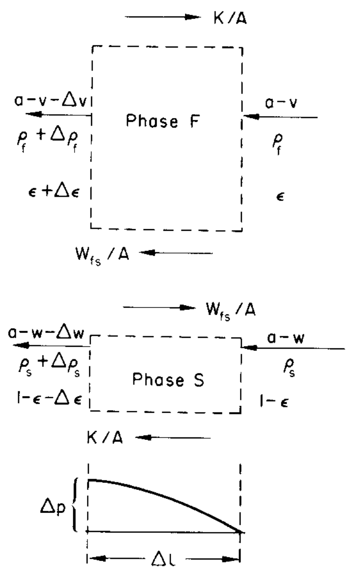


Figure 2. Propagating pressure wave with separate volume elements for both phases.

In this analysis any velocity distributions which might normally occur within the phases S and F are ignored; neither do we consider in detail how, whether by impulses or momentum exchange with the walls or by field forces, a relative velocity v_{rel} between the two phases could be brought about in the otherwise undisturbed motion of the suspension. A constant force K which is equal to the drag force W_{fs} in the steady-state case must act on the volume element of phase S in order to obtain the slip velocity. To establish an equilibrium the same force K must act on the phase F. The pressure drop which is present in any flow in the direction of motion is not involved here. Only the oscillating pressure (amplitude Δp), which is due to the sound wave and which is superimposed on the above mentioned pressure drop, is the cause of the changes considered in velocities, densities and in the porosity. The total drag force W_{fs} must be reduced by the force K so that only the difference in the drag force resulting from the pressure difference Δp appears in the following two momentum balances:

Phase F

$$-(a_{fs} - \Delta v)^2(\rho_f + \Delta\rho_f)(\varepsilon + \Delta\varepsilon) + a_{fs}^2\varepsilon\rho_f - \varepsilon \cdot \Delta p + \frac{W_{fs} - K}{A} = 0. \qquad [7a]$$

Phase S

$$-(a_{fs} + v_{rel} - \Delta w)^2(\rho_s + \Delta\rho_s)(1 - \varepsilon - \Delta\varepsilon) + (a_{fs} + v_{rel})^2(1 - \varepsilon)\rho_s$$
$$-(1 - \varepsilon)\Delta p - \frac{W_{fs} - K}{A} = 0. \qquad [7b]$$

The sum of [7a] and [7b] is the total momentum balance given by [4].

Assuming the suspended particles, droplets, or bubbles of phase S to be of spherical shape and uniform diameter d_p, then the number N of them in the volume element $A \cdot \Delta l$ of the suspension is given by:

$$N = \frac{(1 - \varepsilon) \cdot A \cdot \Delta l}{\pi/6 \cdot d_p^3}. \qquad [8]$$

If the total drag force is equal to the sum of the drag forces acting on the independent and freely moving particles, it follows that:

$$\frac{W_{fs}}{A} = \frac{N}{A} \cdot \frac{1}{2} \cdot c_w \cdot \rho_f \cdot \frac{\pi}{4} \cdot d_p^2 \cdot \frac{1}{\Delta l} \cdot \int_{x=0}^{\Delta l} [v(x) - w(x)]^2 \, dx \qquad [9]$$

and K is given by

$$K/A = (N/A) \cdot \tfrac{1}{2} \cdot c_w \cdot \rho_f \cdot (\pi/4) \cdot d_p^2 \cdot v_{rel}^2. \qquad [10]$$

Due to the sound wave both phases oscillate sinusoidally with the same frequency ν but with different amplitudes. Furthermore, a phase-lag between the two motions exists. If the length of the volume element Δl is chosen just as $\frac{1}{4}$ of the wavelength (figure 2) it follows that within this length the velocities of the phases must vary from $(a - v)$ to $(a - v - \Delta v)$

and from $(a - w)$ to $(a - w - \Delta w)$, respectively, and the following expressions are then valid:

$$\Delta 1 = a_{fs}/4v \tag{11}$$

$$\left.\begin{aligned} v(x) &= a_{fs} - \Delta v \cdot \sin\left(\frac{\pi}{2} \cdot \frac{x}{\Delta 1}\right) \\ w(x) &= a_{fs} + v_{\text{rel}} - \Delta w \cdot \sin\left(\frac{\pi}{2} \cdot \frac{x - s}{\Delta 1}\right). \end{aligned}\right\} \tag{12}$$

Then, if Δv and Δw are small compared with v_{rel}, the following integration yields:

$$\frac{1}{\Delta 1}\int_{x=0}^{\Delta 1} (v - w)^2 \, dx - v_{\text{rel}}^2 = \frac{4}{\pi} v_{\text{rel}} \quad \Delta v - \Delta w \cdot \sqrt{2} \cdot \sin\left[\frac{\pi}{4}\left(1 - \frac{2s}{\Delta 1}\right)\right] \quad . \tag{13}$$

Substituting from [3], [8], [9], [10], [11], and [13] into [7a] and [7b], two linear equations for Δw and Δv are obtained, which lead to the result:

$$\frac{\Delta w}{\Delta v} = \frac{1 + G/\varepsilon}{\dfrac{a_{fs} + v_{\text{rel}}}{a_{fs}} \cdot \dfrac{\rho_s}{\rho_f} + \dfrac{G}{\varepsilon} \cdot \sqrt{2} \cdot \sin\left[\dfrac{\pi}{4}\left(1 - \dfrac{2s}{\Delta l}\right)\right]}, \tag{14}$$

whereas:

$$G = \begin{cases} \dfrac{3}{4\pi} \cdot \dfrac{c_w \cdot v_{\text{rel}}}{v \cdot d_p} = 2 \cdot \dfrac{\rho_s}{\rho_f} \dfrac{g}{w_f \cdot \omega} \cdot \dfrac{v_{\text{rel}}}{w_f}, & \text{[15a]} \\[2ex] \text{or} \quad \dfrac{9}{\pi} \cdot \dfrac{\eta_f}{\rho_f \cdot v \cdot d_p^2} = \dfrac{\rho_s}{\rho_f} \cdot \dfrac{g}{w_f \cdot \omega} \quad \text{when Stokes' law applied.} & \text{[15b]} \end{cases}$$

If Stokes' law applies the calculation is slightly different, since the velocities under the integral in [9] are now linear, resulting in a different value for G (see [15b] above). w_f is the settling velocity of the particles as defined by Rumpf & Gregor (1973), ω is the angular frequency of the sound waves ($\omega = 2\pi v$), and η_f is the dynamic viscosity of the fluid.

In general, the phases do not oscillate with a considerable lag between them, the oscillation of phase S being only slightly behind that of phase F, according to the different inertias. If $s \ll \Delta 1$ or if $\rho_s \gg \rho_f$, then [14] can be written as:

$$\frac{\Delta w}{\Delta v} = \frac{1 + (G/\varepsilon)}{(a_{fs} + v_{\text{rel}})/a_{fs} \cdot (\rho_s/\rho_f) + G/\varepsilon}. \tag{16}$$

With [6] and [16] the velocity of a sound wave in a two-phase suspension can be calculated.

3.1 *Velocity of sound in a solid–gas suspension*

In a two-phase suspension consisting of freely moving solid particles distributed in the continuous gaseous phase, the velocity of sound in the solid particles a_s is very much higher than that in the gas a_f. Furthermore, ρ_s is much greater than ρ_f and the value of ε is close to 1. Therefore in [6] the term $(\rho_f/\rho_s) \cdot (1 - \varepsilon)/\varepsilon \cdot (1/a_s^2)$ can be neglected; it becomes exactly equal to zero, if the solid particles are considered to be incompressible ($a_s \to \infty$). Under this condition, [6] and [16] yield:

$$\left(\frac{a_{fs}}{a_f}\right)^4 \cdot \left\{\frac{\rho_s}{\rho_f}\left(1+\frac{G}{\varepsilon}-G\right)+G\right\}+\left(\frac{a_{fs}}{a_f}\right)^3 \cdot \left\{2\cdot\frac{\rho_s}{\rho_f}\left(1+\frac{G}{\varepsilon}-G\right)+G\right\}\cdot\frac{v_{\text{rel}}}{a_f}$$
$$+\left(\frac{a_{fs}}{a_f}\right)^2 \cdot \left\{\frac{\rho_s}{\rho_f}\left(\frac{v_{\text{rel}}}{a_f}\right)^2 \cdot \left(1+\frac{G}{\varepsilon}-G\right)+1-\frac{\rho_s}{\rho_f}-\frac{1}{\varepsilon}-\frac{G}{\varepsilon^2}\right\}$$
$$-\frac{a_{fs}}{a_f}\cdot\frac{v_{\text{rel}}}{a_f}\cdot\left\{2\cdot\frac{\rho_s}{\rho_f}+\frac{G}{\varepsilon}\right\}-\left(\frac{v_{\text{rel}}}{a_f}\right)^2\cdot\frac{\rho_s}{\rho_f}=0. \qquad [17]$$

For a constant ratio of densities, $\rho_s/\rho_f = 2000$, the solution of [17], viz. the sound velocity in the suspension a_{fs} related to the sound velocity in the pure gas a_f, as a function of the porosity ε the relative velocity v_{rel}/a_f and the parameter G, is plotted in figure 3 for values of $v_{\text{rel}}/a_f = 0, 0.030, 0.076, 0.152$ corresponding to $v_{\text{rel}} = 0, 10, 25, 50$ m/s respectively, and $a_f = 330$ m/s in all cases.

Since the curves in figure 3 do not vary much with respect to v_{rel}, the relative sound velocity a_{fs}/a_f is plotted in figure 4 without regard to the influence of v_{rel}, i.e. the curves are calculated by putting $v_{\text{rel}} = 0$ in [17]. It must be pointed out that only the direct influence of v_{rel} in [17] is neglected, since the parameter G also contains v_{rel} (see [15a]) and therefore the value of v_{rel} still determines the sound velocity. Several special cases have to be considered:

(1) If the value of the particle size or of the frequency approaches zero ($d_p \to 0$ or $\nu \to 0$), then [15] and [16] yield:

$$G \to \infty, \qquad \frac{\Delta w}{\Delta v} = 1,$$

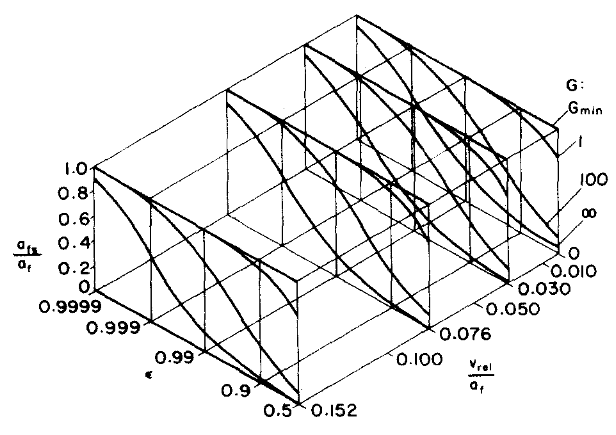


Figure 3. Velocity of sound in a solid–gas suspension depending on parameter G, porosity ε, and relative velocity v_{rel} (density-ratio $\rho_s/\rho_f = 2000$).

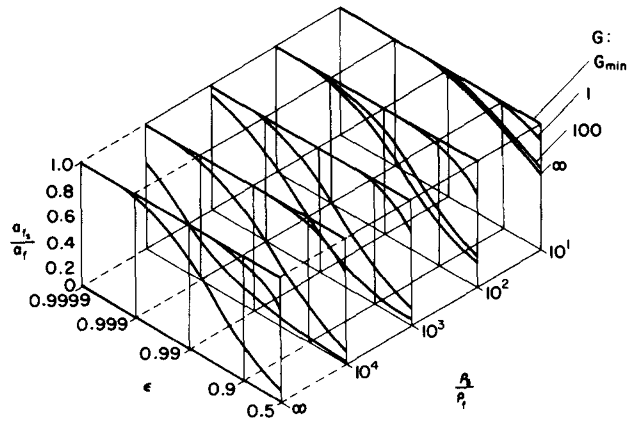


Figure 4. Velocity of sound in a solid–gas suspension depending on parameter G, porosity ε, and density-ratio ρ_s/ρ_f (influence of v_{rel} neglected).

i.e. the sound velocity has the lowest possible values, as already shown by Rumpf & Gregor (1973).

(2) If the ratio of the densities increases to infinity ($\rho_s/\rho_f \to \infty$), [16] yields: $\Delta w/\Delta v = 0$ and hence:

$$\left.\frac{a_{fs}}{a_f}\right|_{\rho_s/\rho_f \to \infty} = \frac{1}{\sqrt{1 + \dfrac{1-\varepsilon}{\varepsilon} \cdot G}} = \begin{cases} 0 & \text{if } G \to \infty \\ 1 & \text{if } G = 0. \end{cases} \qquad [18]$$

The velocity of sound decreases with decreasing particle size, with decreasing frequency, and with increasing ratio of densities. Only if (i) the value of the particle size or of the frequency becomes zero and if (ii) at the same time the ratio of the densities rises to infinity, does the value of the sound velocity become zero. In this case the propagation of a sonic wave would no longer be possible, because the infinitely small particles would oscillate with the same amplitude as the gas ($\Delta w/\Delta v = 1$, see item 1 above) but, due to their infinitely large mass, the whole sonic energy would be dissipated.

(3) Compared with the pure gas, the solid particles in a solid–gas suspension displace gas volume elements and thus, during the passage of a pressure wave, they are subjected to a momentum which cannot be less than the one which would act on the displaced gas volume elements. This momentum results in a change of velocity, Δw, which therefore cannot become arbitrarily small.

The velocity of sound in the suspension must always be smaller than in the pure gas. This condition, $a_{fs}/a_f \leq 1$, when applied to [6], leads to the inequality:

$$\frac{1}{(v_{rel}/a_f + 1) \cdot \dfrac{\rho_s}{\rho_f} - \dfrac{1}{\varepsilon(v_{rel}/a_f + 1)}} \leq \frac{\Delta w}{\Delta v} \leq 1. \qquad [19]$$

Hence, from [16] and [19], the limits for the parameter G may be obtained:

$$G_{\min} = \frac{1}{\dfrac{\rho_s}{\rho_f}\left(1 + \dfrac{v_{\text{rel}}}{a_f}\right)^2 - \dfrac{v_{\text{rel}}}{a_f} - \dfrac{1+\varepsilon}{\varepsilon}} \leq G < \infty. \quad [20]$$

Therefore in figures 3 and 4 the parameter G varies between $G_{\min}$ and ∞.

It should be noticed that, whenever both phases flow without any relative velocity ($v_{\text{rel}} = 0$), and have the same accelerations ($\Delta w/\Delta v = 1$ or $G \to \infty$), the velocity of sound is then given by:

$$\frac{a_{fs}^2}{a_f^2} = \frac{1}{\varepsilon \cdot [\varepsilon + (1-\varepsilon)\cdot(\rho_s/\rho_f)]}.$$

This equation may also be obtained by applying Laplace's equation for the propagation of sound waves in a two-phase media, as has been shown by Hinrichs (1965), Pfleiderer (1957), and Gouse & Brown (1964).

3.2 *Velocity of sound in a liquid–gas suspension*

If liquid droplets are suspended homogeneously in a continuous gaseous phase, [17] or figures 3 and 4 can be used for determining the velocity of sound, if the subscript "s" for solid is substituted by "l" for liquid. The subscript "f" now stands for the gas. It must be made sure that the expression, $(\rho_f/\rho_s)\cdot(1-\varepsilon)/\varepsilon\cdot(1/a^2)$, in [6] is still negligible compared with $1/a_f^2$, otherwise the complete [6] and [16] have to be applied.

If the porosity ε becomes small, say $\varepsilon < 0.7$, then the liquid phase can no longer be considered as freely moving droplets distributed in a continuous gaseous phase (spray flow). A wavy or stratified flow, also a slug or even plug flow is obtained with smaller ε-values, making the application of the given theory impossible. In the case of bubble flow, the theory can be used, but it needs some modifications, as will now be explained.

The continuous phase is now the liquid phase, and the gaseous phase is distributed homogeneously as bubbles. It is more useful to relate the velocity of sound a_{fl} in the liquid–gas suspension to the motion of the liquid phase (L):

$$a_{fl} = a - w. \quad [21]$$

Since $a_{fs} = a_{fl} - v_{\text{rel}}$ the following relation is obtained instead of [6]:

$$a_{fl}^2\left(\frac{1}{a_f^2} + \frac{\rho_f}{\rho_l}\cdot\frac{1-\varepsilon}{\varepsilon}\cdot\frac{1}{a_l^2}\right) = \frac{1 + (1-\varepsilon)/\varepsilon\cdot(a_{fl} - v_{\text{rel}})/a_{fl}\cdot(\Delta w/\Delta v)}{\varepsilon(1 + (1-\varepsilon)/\varepsilon\cdot a_{fl}/(a_{fl} - v_{\text{rel}})\cdot(\rho_l/\rho_f)\cdot\Delta w/\Delta v)}\cdot\left(\frac{a_{fl}}{a_{fl} - v_{\text{rel}}}\right)^2 \quad [22]$$

and instead of [16]:

$$\frac{\Delta w}{\Delta v} = \frac{(a_{fl} - v_{\text{rel}})/a_{fl}\cdot(\rho_f/\rho_l) + G/(1-\varepsilon)}{1 + G(1-\varepsilon)} \quad [23]$$

$$G = \begin{cases} \dfrac{3}{4\pi} \cdot \dfrac{c_w \cdot v_{rel}}{\nu \cdot d_p} & \\ \dfrac{9}{\pi} \cdot \dfrac{\eta_l}{\rho_l \cdot \nu \cdot d_p^2} & \text{Stokes' law.} \end{cases} \qquad [24]$$

Here d_p is the bubble diameter and η_l is the viscosity of the liquid phase. Two special cases are considered:

(1) $G \to \infty$, i.e. bubble diameter $d_p = 0$, or frequency $\nu = 0$ yields:

$$\Delta w/\Delta v = 1.$$

Since in this case the number of bubbles is infinite for a given ε-value, the sound velocity is the lowest.

(2) If the ratio of the densities increases to infinity ($\rho_l/\rho_f \to \infty$), the result of [22] is:

$$a_{fl} = v_{rel}.$$

Because in the continuous liquid phase no sound propagates due to its infinite density, the sound is transported only by the compressible gas bubbles moving with a velocity of v_{rel} relative to the liquid.

An example for the velocity of sound in a liquid–gas suspension, covering the whole porosity range from $\varepsilon = 0$ to $\varepsilon = 1$, is shown in figure 5. The curves are calculated using [6] with $v_{rel} = 0$:

$$a_{fl}^2 = \frac{1 + \dfrac{1-\varepsilon}{\varepsilon} \cdot \dfrac{\Delta w}{\Delta v}}{\varepsilon\left(1 + \dfrac{1-\varepsilon}{\varepsilon} \cdot \dfrac{\rho_l}{\rho_f} \cdot \dfrac{\Delta w}{\Delta v}\right) \cdot \left(\dfrac{1}{a_f^2} + \dfrac{\rho_f}{\rho_l} \cdot \dfrac{1-\varepsilon}{\varepsilon} \cdot \dfrac{1}{a_l^2}\right)} \qquad [25]$$

$$\frac{\Delta w}{\Delta v} = \begin{cases} \dfrac{1 + (G/\varepsilon)}{(\rho_l/\rho_f) + (G/\varepsilon)} & \text{according to [16] if } 0.5 < \varepsilon \leq 1 \\ \dfrac{(\rho_f/\rho_l) + G/(1-\varepsilon)}{1 + G/(1-\varepsilon)} & \text{according to [23] if } 0 \leq \varepsilon < 0.5. \end{cases}$$

As mentioned above the present theory is not valid in the middle porosity range because neither phase can be regarded as dispersed in the other.

Only when both phases have the same acceleration ($\Delta w/\Delta v = 1$, i.e. $G \to \infty$) are continuous curves obtained for the whole porosity range. This is possible if the size of the droplets or bubbles of the suspended phase is infinitely small ($d_p = 0$) or if the densities are equal ($\rho_f = \rho_l$). Equation [25] can then be written as:

$$\frac{1}{a_{fl}^2} = \frac{\varepsilon}{a_f^2}\left[1 + (1-\varepsilon)\left(\frac{\rho_l}{\rho_f} - 1\right)\right] + \frac{1-\varepsilon}{a_l^2} \cdot \left[1 + \varepsilon\left(\frac{\rho_f}{\rho_l} - 1\right)\right].$$

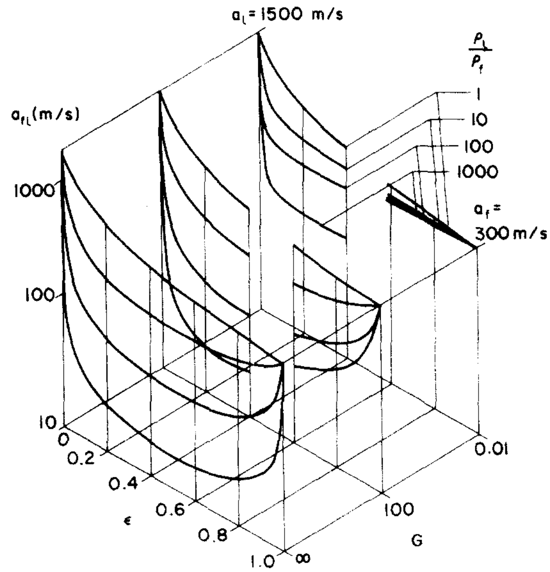


Figure 5. Velocity of sound in a liquid–gas suspension depending on parameter G, porosity ε, and density-ratio ρ_l/ρ_f.

By applying Laplace's equation to a two-phase system this expression was also obtained by Böckh & Chawla (1972).

3.3 *Velocity of sound in solid–gas systems of low porosity*

For a solid–gas mixture with a low porosity it is no longer possible to apply the given theory, because the gas bubbles are not freely moving in the solid, continuous phase. The elastic behaviour of loosely packed particles such as sand is very different from that of bulky porous materials such as sintered metals. The velocity of sound in such media can also be influenced by the packing structure, particle- and pore-size distribution, and by the type of bonding between the particles. So far no theory exists which considers all these factors.

A simple theory has been put forward by Wyllie, Gregory & Gardner (1956), which yields an equation for the sonic velocity by means of a linear interpolation, with respect to the porosity between the travel time of a sound wave in the pure solid material and that in the pure fluid. Wachholz (1962) has improved this theory with a cubic interpolation in which the necessary constants are chosen so as to fit the experimental data.

Biot's theory (1955) is very much more refined, but assumes the density of the fluid saturating the pores to be similar to the density of the solid, Geertsma & Smit (1961) apply this theory for geophysical purposes.

White & Sengbush (1953) present a theory for determining the sonic velocity in a simple cubic packing of spherical particles, but their equation is obtained for the seismic velocity at shallow depths in the earth with all the consequent restrictions. Another similar theory, put forward by Meister (1968), is only valid for two values of the porosity and does not show clearly what the effect of the porosity is.

The present investigations are confined, on the one hand, to sintered materials containing gas-pores and on the other to packed particles.

Gregor (1971) compared the dynamically measured Young's modulus of sintered materials with the statically measured Young's modulus. Since it could be shown that they are identical (with certain small restrictions which have to be made also in case of non-porous materials), the sonic velocity in a sintered material can be obtained by the same formula as for a non-porous material:

$$a_{fs} = \sqrt{E_{fs}/\rho_{fs}}. \tag{26}$$

E_{fs} is the Young's modulus of the porous material, and ρ_{fs} is its density, being equal to $(1 - \varepsilon) \cdot \rho_s$.

Comparing the different theories for the calculation of E_{fs}, Gregor (1971) found that the theory of Skorokhod (1963) was the most useful one. His result can be approximated by the formula

$$E_{fs}/E_s = 1 - 2 \cdot \varepsilon. \tag{27}$$

(E_s = Young's modulus of the non-porous material.)

Equation [26] yields with [27]:

$$\frac{a_{fs}}{a_s} = \sqrt{\frac{1 - 2 \cdot \varepsilon}{1 - \varepsilon}}. \tag{28}$$

The dependence of E_{fs} on the particle size has not been investigated systematically yet although Artusio *et al.* (1966) have mentioned the decrease of Young's modulus with increasing particle size. According to the sinter theory, in the first phase of the sintering procedure the particle diameter d_p is proportional to b^a, where b is the neck diameter and $a > 2$. Therefore a sintered material with a larger particle size is less resistant to a deformation than one with a smaller particle size. Thus Young's modulus and also the velocity of sound would decrease with bigger particles but no quantitative result, theoretical or experimental, has yet been obtained.

A curve for E_{fs} which fits very well has been experimentally obtained by McAdam (1951):

$$E_{fs}/E_s = (1 - \varepsilon)^{3.4}. \tag{29}$$

This equation not only fits his own experimental values but also those of other authors like Tschechowa & Franzewitsch (1958) or Knudsen (1962). The combined result of [26] and [29] is the following experimentally found relationship:

$$a_{fs}/a_s = (1 - \varepsilon)^{1.2}. \tag{30}$$

For determining the sonic velocity in packed materials such as sand, a simple model of a chain of equal spheres lying beside each other is chosen. If this chain is compressed elastically the stress–strain diagram gives the parabola $\sigma = C \cdot (\Delta 1/1)^{3/2}$ according to Hertz's equation for the deformation of spheres. Since this curve has a horizontal tangent in the origin, Young's modulus at the origin is equal to zero and so is the sonic velocity. In connection with their theoretical and experimental investigations at shallow depths in the earth White & Sengbush (1953) have already stated this fact; they found a proportionality between the sonic velocity and the sixth root of the depth in the earth.

In packed materials the bonding mechanism within the agglomerate is provided by the liquid bridges between the particles and by van der Waal's forces, the latter being negligibly small, as shown by Herrmann (1971). Due to these forces a certain preloading on the particles is always present in an agglomerate and, according to Schubert, Herrmann & Rumpf (1974), this force is almost equal to the tensile stress, σ_t, of the agglomerate. Since the applied pressure in a sound wave is very much smaller than the tensile stress of an agglomerate, Young's modulus can be defined as the derivative of the above-mentioned parabola of Hertz at the point when $\sigma = \sigma_t$, the tensile stress. Schubert, Herrmann & Rumpf (1974) obtained the result:

$$E_{fs} = \frac{3}{2} \cdot \left[\frac{E_s^2 \cdot \sigma_t \cdot (1 - \varepsilon)^2}{9 \cdot (1 - v_s^2)^2 \cdot \varepsilon^2} \cdot \frac{2r}{d_p}\right]^{1/3} \qquad [31]$$

where v_s is the Poisson-ratio of the non-porous material, d_p is the particle diameter, and r is the radius of curvature (at the contact point), which is usually much smaller than the particle radius, $d_p/2$, due to the rough particle surface.

Equation [31] yields with [26]:

$$\frac{a_{fs}}{a_s} = \left[\frac{3}{8} \cdot \frac{\sigma_t}{E_s \cdot (1 - v_s^2)^2} \cdot \frac{1}{\varepsilon^2 (1 - \varepsilon)} \cdot \frac{2r}{d_p}\right]^{1/6}. \qquad [32]$$

Because in the model used the agglomerate consists of spherical particles of equal size, [32] is not valid for $\varepsilon < 0.26$. In the case of non-uniform particles, the porosity can take even smaller values, so that [32] can only show the tendency and probably needs to be modified.

In figure 6 for sintered materials [28] is compared with the experimental results given by [30].

The proposed theory for the velocity of sound in packed materials resulting in [32], different curves depending on the parameter

$$\frac{E_s(1 - v_s^2)^2}{\sigma_t} \cdot \frac{d_p}{2r}$$

are obtained and also shown in figure 6.

Comparison between theoretical and experimental results

For checking the applicability of the theory, the theoretical results are compared in figure 7 with experimental values obtained from a few sources in the literature where sufficient data about the experiments was given.

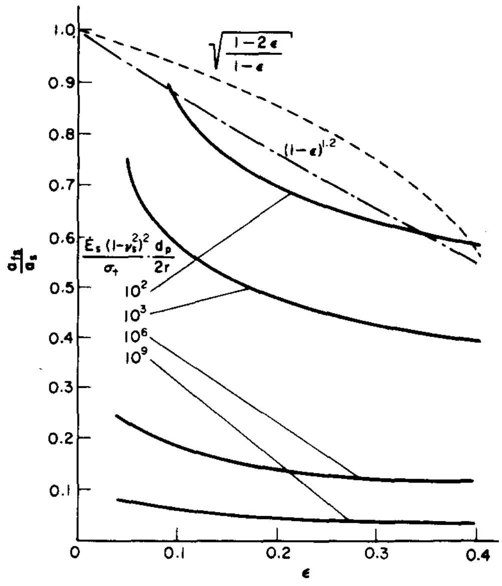


Figure 6. Theoretical and experimental results for the velocity of sound in solid–gas media with low porosity.

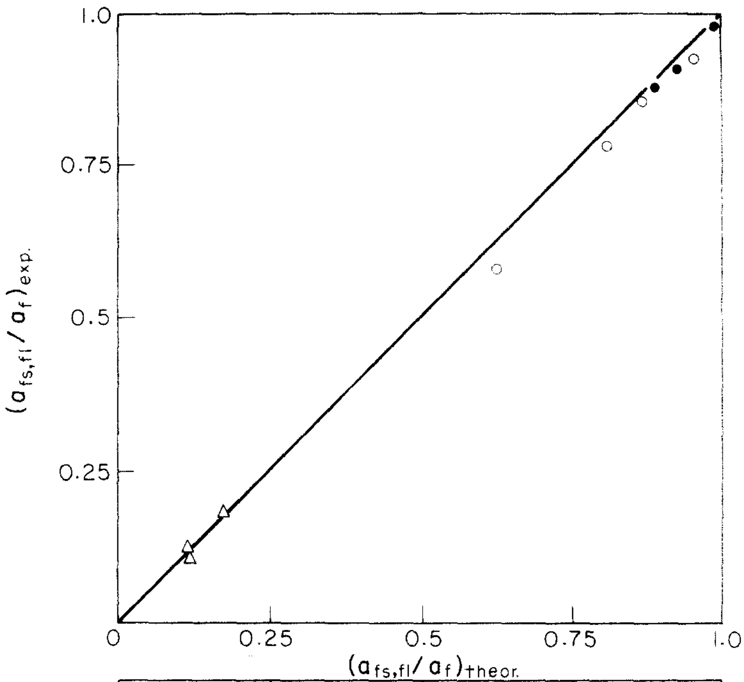
In the case of solid–gas suspension (high porosity range) the results of [15] and [17] are compared with experiments carried out by Zink & Delasso (1958) and also reported by Temkin & Dobbins (1966). A comparison has also been made with the results of Soo (1967).

Since in the theory the phase-lag between the two phases has been neglected, [16] gives the smallest possible values for $\Delta w/\Delta v$ and therefore the theory always yields higher values than the experiments do.

For liquid–gas suspensions, the data obtained by Böckh & Chawla (1973) fit well with the theory if the bubble or droplet size is put equal to zero, i.e. $G \to \infty$. Due to experimental difficulties the determination of d_p was not possible.

For checking [32] no experimental data could be found, but an example is given here, based on some yet unpublished work:

According to Meister (1968) Young's modulus of quartz is equal to $E_s = 7.8 \times 10^6$ N/cm^2 and the sound velocity in quartz is $a_s = 5700$ m/s. Assuming the particles to be smooth with $d_p = 2r$, and $\sigma_t = 0.1$ N/cm^2, which latter figure, according to Schubert, Herrmann & Rumpf (1974), is a reasonable value for "dry" agglomerates, the sound velocity in sand



Symbol	Author	Phase F	Phase S,L
○	Zink, Delasso	Air	Al_2O_3
●	Soo	Air	Mg
△	v. Böckh, Chawla	Air	Water

Figure 7. Comparison between theoretical and experimental results.

with a porosity of $\varepsilon = 0.43$ is found from [32] to be $a_{fs} \approx 500$ m/s instead of $a_{fs} = 370$ m/s, the experimentally determined value of Hunter & Matsukawa (1961). More experimental work in this field is necessary to check the proposed theory.

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Résumé—Une théorie est développée afin de calculer la vitesse du son dans des systèmes solide-gazeux et liquide-gazeux. En équilibrant les masses et les moments, on obtient une équation montrant que la vitesse du son dépend de la vitesse de glissement entre les deux phases, du rapport des densités, de la porosité, du diamètre de la particule, du coefficient de drainage, et de la fréquence du son.

La théorie ne peut être appliquée que si une phase est distribuée d'une manière homogène dans l'autre sous forme de particules, de gouttes ou de bulles de dimensions égales. Les résultats expérimentaux confirment très bien la théorie.

Fondée sur l'equation de Hertz de la déformation sphérique, une autre théorie est décrite permettant de calculer la vitesse du son dans des systèmes gazeux–solides de faible porosité, tels que des enveloppes ou des corps poreux.

Auszug—Eine Théorie zur Berechnung der Schallgeschwindigkeit in Feststoff–Gas bzw. Flüssigkeits–Gas Gemischen wird mit Hilfe von Massen- und Impulsstrombilanzen abgeleitet. Die Schallgeschwindigkeit hängt ab von der Geschwindigkeitsdifferenz zwischen beiden Phasen, vom Dichteverhältnis, von der Porosität, vom Teilchendurchmesser, vom Widerstandskoeffizienten und von der Schallfrequenz.

Die Theorie kann dann angewendet werden, wenn die eine Phase in der anderen homogen als Teilchen, Tropfen oder Blasen gleicher Größe verteilt ist. Experimente zeigen eine sehr gute Übereinstimmung mit der Theorie.

Ferner wird eine Theorie angegeben, die auf der Hertz' schen Abplattung von Kugeln beruht und die es erlaubt, die Schallgeschwindigkeit in Feststoff–Gas Systemen mit niedrigen Porositäten, wie z.B. in Haufwerken oder porösen Körpern, zu berechnen.

Резюме—Развита теория для рассчета скорости звука в системах твердое теложидкости и газ-жидкость. С помощью балансы масс и моментов, было получено уравнение, показывающее, что скорость звука зависит от скорости проскальзывания между двумя фазами, от соотношения диаметров, от пористости, от коэфициента запаздывания и от частотн звука.

Излагаемая теория может быть применена только в том случае, если одна фаза распред; елена в другой в виде частиц, капелек или пузырьков равного размера гомогенном образом. Раезультатн эксперимента показывают очень хорошее согласие с данной теорией.

Описано дальнейшее развитие теории, основанное на уравнении Герца дл я деформации сферы, что позволяет рассчитать скорость звука в малопористох газо-твердьх системах, как нацример набивки или пористые тала.